NAG Toolbox for MATLAB

g01jc

1 Purpose

g01jc returns the lower tail probability of a distribution of a positive linear combination of χ^2 random variables.

2 Syntax

```
[p, pdf, ifail] = g01jc(a, mult, rlamda, c, tol, 'n', n, 'maxit', maxit)
```

3 Description

For a linear combination of noncentral χ^2 random variables with integer degrees of freedom the lower tail probability is

$$P\left(\sum_{j=1}^{n} a_j \chi^2(m_j, \lambda_j) \leq c\right), \tag{1}$$

where a_j and c are positive constants and where $\chi^2(m_j, \lambda_j)$ represents an independent χ^2 random variable with m_j degrees of freedom and noncentrality parameter λ_j. The linear combination may arise from considering a quadratic form in Normal variables.

Ruben's method as described in Farebrother 1984 is used. Ruben has shown that (1) may be expanded as an infinite series of the form

$$\sum_{k=0}^{\infty} d_k F(m + 2k, c/\beta), \tag{2}$$

where $F(m + 2k, c/\beta) = P(\chi^2(m + 2k) < c/\beta)$, i.e., the probability that a central χ^2 is less than c/β.

The value of β is set at

$$\beta = \beta_B = \frac{2}{(1/a_{\min} + 1/a_{\max})}$$

unless $\beta_B > 1.8 a_{\min}$, in which case

$$\beta = \beta_A = a_{\min}$$

is used, where $a_{\min} = \min\{a_j\}$ and $a_{\max} = \max\{a_j\}$, for $j = 1, 2, \ldots, n$.

4 References

Farebrother R W 1984 The distribution of a positive linear combination of χ^2 random variables *Appl. Statist.* **33 (3)**

5 Parameters

5.1 Compulsory Input Parameters

1: $\mathbf{a}(\mathbf{n})$ – **double array**

The weights, $a_1, a_2, \ldots, a_n$.

Constraint: $\mathbf{a}(i) > 0.0$, for $i = 1, 2, \ldots, n$.

2: **mult(n) – int32 array**

The degrees of freedom, $m_1, m_2, \ldots, m_n$.

Constraint: $\textbf{mult}(i) \geq 1$, for $i = 1, 2, \ldots, \textbf{n}$.

3: **rlamda(n) – double array**

The noncentrality parameters, $\lambda_1, \lambda_2, \ldots, \lambda_n$.

Constraint: $\textbf{rlamda}(i) \geq 0.0$, for $i = 1, 2, \ldots, n$.

4: **c – double scalar**

c, the point for which the lower tail probability is to be evaluated.

Constraint: $\textbf{c} \geq 0.0$.

5: **tol – double scalar**

The relative accuracy required by you in the results. If g01jc is entered with **tol** greater than or equal to 1.0 or less than 10 times the ***machine precision*** (see x02aj), then the value of 10 times ***machine precision*** is used instead.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the arrays **a**, **mult**, **rlamda**. (An error is raised if these dimensions are not equal.)

n, the number of χ^2 random variables in the combination, i.e., the number of terms in equation (1).

Constraint: $\textbf{n} \geq 1$.

2: **maxit – int32 scalar**

The maximum number of terms that should be used during the summation.

Constraint: $\textbf{maxit} \geq 1$.

5.3 Input Parameters Omitted from the MATLAB Interface

wrk

5.4 Output Parameters

1: **p – double scalar**

The lower tail probability associated with the linear combination of n χ^2 random variables with m_j degrees of freedom, and noncentrality parameters λ_j, for $j = 1, 2, \ldots, n$.

2: **pdf – double scalar**

The value of the probability density function of the linear combination of χ^2 variables.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g01jc may return useful information for one or more of the following detected errors or warnings.

If on exit $\textbf{ifail} = 1$ or 2, then g01jc returns 0.0.

ifail = 1

On entry, **n** < 1,
or **maxit** < 1,
or **c** < 0.0.

ifail = 2

On entry, **a** has an element ≤ 0.0,
or **mult** has an element < 1,
or **rlamda** has an element < 0.0.

ifail = 3

The central χ^2 calculation has failed to converge. This is an unlikely exit. A larger value of **tol** should be tried.

ifail = 4

The solution has failed to converge within **maxit** iterations. A larger value of **maxit** or **tol** should be used. The returned value should be a reasonable approximation to the correct value.

ifail = 5

The solution appears to be too close to 0 or 1 for accurate calculation. The value returned is 0 or 1 as appropriate.

7 Accuracy

The series (2) is summed until a bound on the truncation error is less than **tol**. See Farebrother 1984 for further discussion.

8 Further Comments

None.

9 Example

```
a = [6;
     3;
     1];
mult = [int32(1);
     int32(1);
     int32(1)];
rlamda = [0;
     0;
     0];
c = 20;
tol = 0.0001;
[p, pdf, ifail] = g01jc(a, mult, rlamda, c, tol)
```

```
p =
    0.8760
pdf =
    0.0129
ifail =
           0
```